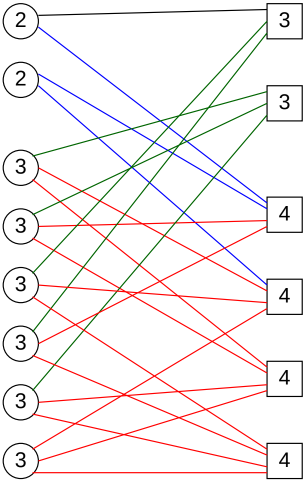
Detailed Evolution of Degree Distributions in Residual Graphs with Joint Degree Distribtuions

Takayuki Nozaki [1], Kenta Kasai [1],
Tomoharu Shibuya [2] and Kohichi Sakaniwa [1]

[1] Tokyo Institute of Technology, Tokyo
[2] Sophia University, Tokyo

July 9th, 2008

Detailedly represented irregular LDPC code ensembles [Kasai, et al. '03]



○: variable node

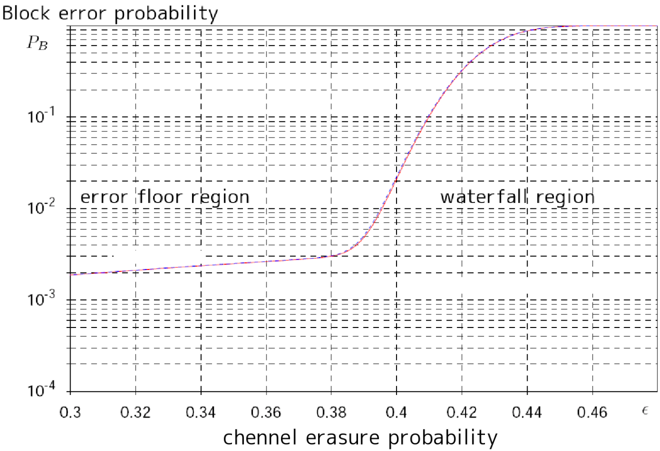
□: check node

$\pi(i,j)$: joint degree distribution

$\pi(2,3) = \frac{1}{22}$, $\pi(2,4) = \frac{3}{22}$

$\pi(3,3) = \frac{5}{22}$, $\pi(3,4) = \frac{13}{22}$

Finite-length scaling [Amraoui, et al. '03]



(3,6)-regular, block-length 2048

Finite-length optimization [Amraoui, et al. '06]

Optimized degree distribution pair

$$\lambda(x) = 0.0739196x + 0.657891x^2 + 0.268189x^{12}$$
$$\rho(x) = 0.390753x^4 + 0.361589x^5 + 0.247658x^9$$

- channel erasure probability 0.5
- blocklength 5000
- target block error probability 10^{-4}

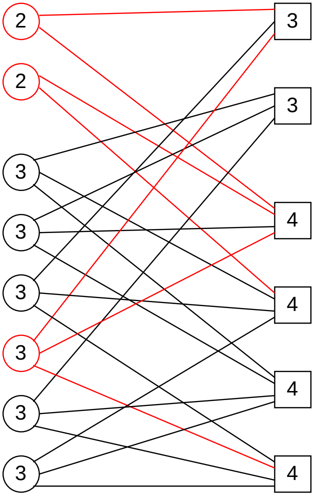
Our goal

- Finite-length scaling with joint degree distributions.

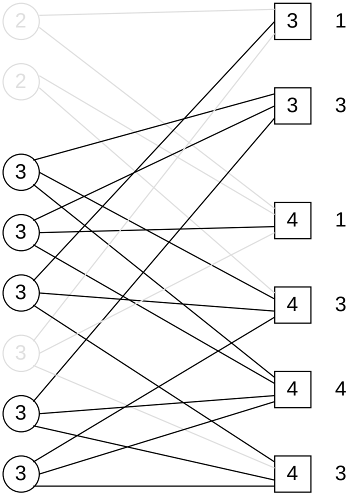
Our goal

- Finite-length scaling with joint degree distributions.
- As a first step, we analyze an iterative decoder.

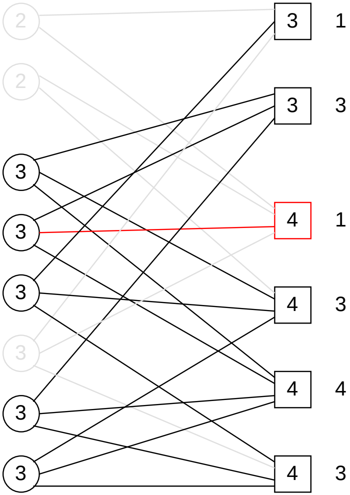
Peeling Algorithm



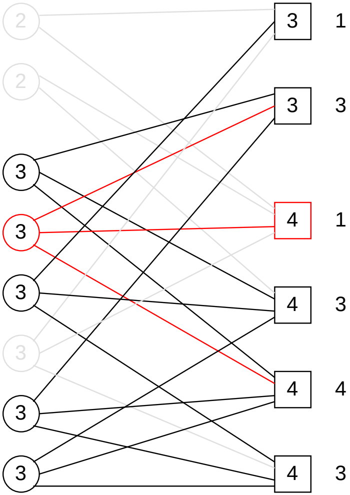
Peeling Algorithm



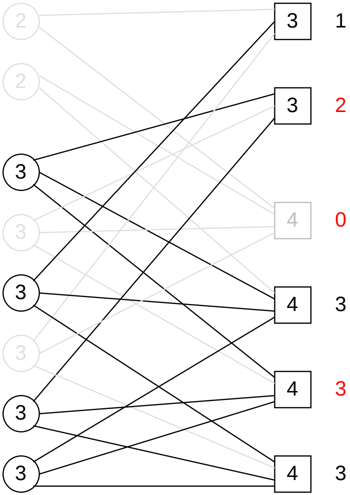
Peeling Algorithm



Peeling Algorithm



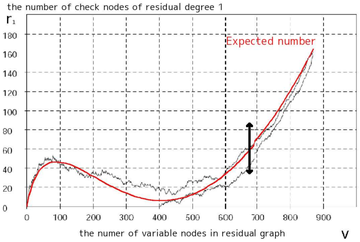
Peeling Algorithm

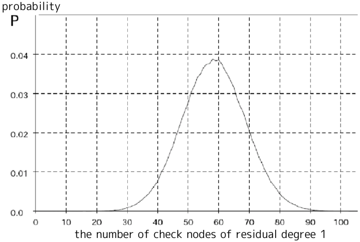


Distribution of the number of check nodes of residual degree one [Amraoui, et al. '03]

Amraoui, et al '03

The ditribution of the number of check nodes of residual graph tends to a Gaussian.





(3,6)-regular ensemble, block-length 2048, channel erasure probability 0.425

Expected number of check nodes of residual degree 1

theorem1

For detailedly represented ensembles, $r_{s,1}$ are written as

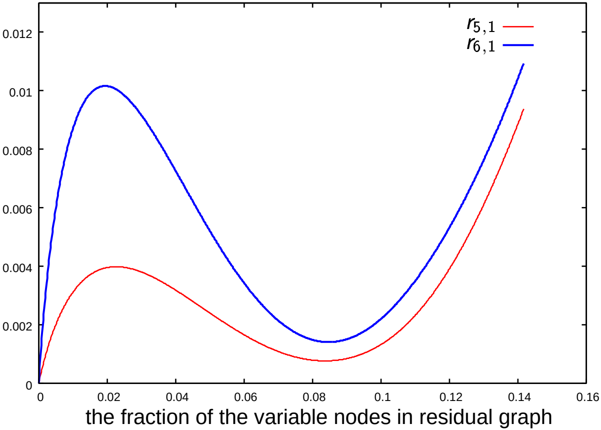
$$r_{s,1} = \epsilon \sum_{i=2}^{\mathrm{d_l}} \pi(i,s) \left(\sum_{j=2}^{\mathrm{d_r}} \rho_i(j) y_j \right)^{i-1} \left[y_s - 1 + \left\{ 1 - \epsilon \sum_{i=2}^{\mathrm{d_l}} \lambda_s(i) \left(\sum_{j=2}^{\mathrm{d_r}} \rho_i(j) y_j \right)^{i-1} \right\}^{s-1} \right].$$

where

$$r_{s,1} := \frac{\text{(the number of check nodes of residual degree 1 and original degree } s\text{)}}{\text{(the total number of the edges in Tannar graph)}}$$

$\mathbf{y} := (y_2, y_3, \dots, y_{\mathrm{d_r}})$ is given as a solution of differential equations

Expected number of check nodes of residual degree 1



$$r_{s,1} := \frac{(\text{the number of check nodes of residual degree 1 and original degree } s)}{(\text{the total number of edges})}$$

Graphical interpretation of successful decoding

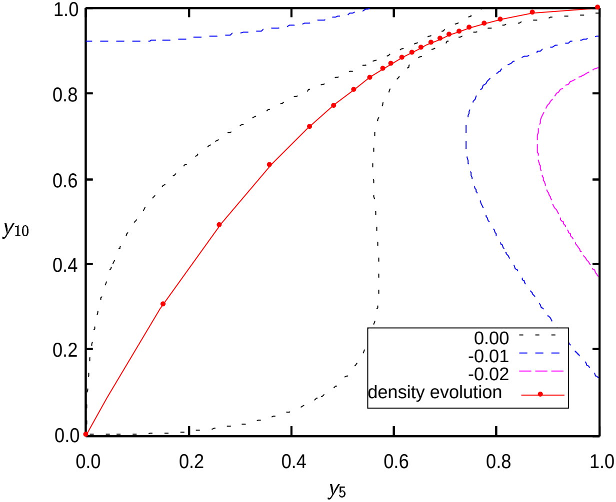
theorem2

A path from the point $(1, 1, \ldots, 1)$ to $(0, 0, \ldots, 0)$ exist in the set $\{\mathbf{y} \mid \min_s r_{s,1}(\mathbf{y}) > 0\}$

$\Longleftrightarrow$

the decoding is successful.

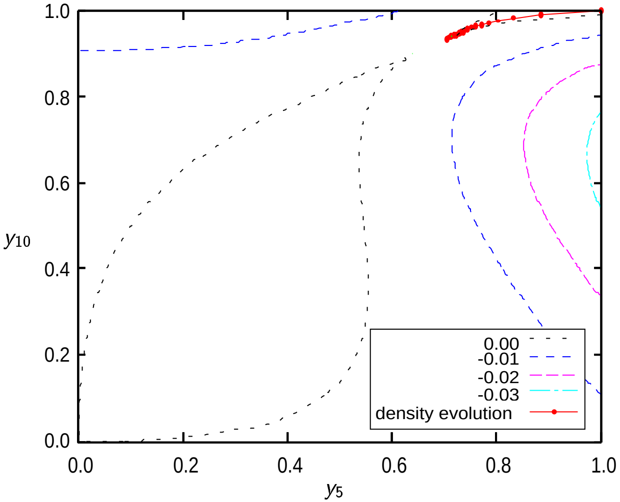
Graphical interpretation of successful decoding



$\epsilon = 0.40$, $\pi(3,5) = 0.12, \pi(3.10) = 0.18, \pi(4,5) = 0.28, \pi(4.10) = 0.42$

The decoding is successful.

Graphical interpretation of successful decoding



$\epsilon = 0.415$, $\pi(3,5) = 0.12, \pi(3.10) = 0.18, \pi(4,5) = 0.28, \pi(4.10) = 0.42$

The decoding is not successful.

Conclusion and Future work

Conclusion

- We derive the expected number of check nodes of residual degree 1
- We give a graphical interpretation of sucessful decoding

Future work

- derive the variance of the number of check nodes of residual degree 1
- analyze the waterfall region with joint degree distributions
- optimize detailedly represented ensembles

bibliography

[1] K. Kasai, T. Shibuya and K. Sakaniwa, "Detailedly Represented Irregular Low-Density Parity-Check Codes," *IEICE Trans. on Fundamentals*, vol.E86-A, no.10, pp.2435-2444, 2003.

[2] A. Amraoui "Asymptotic and finite-length optimization of LDPC codes," Ph.D. Thesis, EPFL, June 2006.

[3] M. Luby, M. Mitzenmacher, A. Shokrollahi, D. A. Spielman, and V. Stemann. "Practical Loss-Resilient Codes," in *Proceedings of the 29th annual ACM Symposium on Theory of Computing*, 1997, pp. 150-159.